Teacher-Orchestrated Classroom Arguments

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One great challenge of the reform inspired by NCTM's Standards (1989, 2000) has been to transform classroom discourse in ways that actively involve students and use their ideas as sources of mathematical ideas in the classroom community. In this article, I present a type of classroom discourse—teacher-orchestrated classroom argument—that serves those functions.

Given the historical resilience of traditional teaching practices (Jacobs et al. 2006; Stigler and Hiebert 1999), it is vital to articulate new practices as well as demonstrate how teachers come to enact such practices. Teacher-orchestrated classroom argument is one such practice.

A VISION OF REFORM DISCOURSE

This specific vision of reform discourse expects that students should engage in mathematical argumentation with the strong guidance of the teacher. Forman (2003) describes teacher-orchestrated classroom arguments as having two main characteristics: (1) the teacher plays a strong role in initiating students into the practice of mathematical argumentation; and (2) through the practice of mathematical argumentation, students learn to engage intellectually with others' ideas by reflecting and building on the mathematical explanations of their peers. The term *argument* should not be construed as a competition to decide whose answer is best; rather, it should imply a deliberate negotiation of a common explanation, one that supersedes and incorporates individuals' explanations.

Forman (2003) states that in teacher-orchestrated classroom arguments, teachers "recruit students, align students with positions in the collective argument, attribute attitudes and intentions to students, and report or clarify students' explanations" (p. 344). Below, I describe these teacher actions and how they function to engage students in mathematical argumentation.

Recruiting students into a discussion involves eliciting explanations in different activity settings. One approach is to call on students for solutions in a whole-class setting without having previewed their solutions. A subtler approach requires some prior knowledge of students' solutions, observed, for example, during group work. A teacher who observes student strategies in individual or group work can strategically call on those students whose solutions provide critical illustrations or contrasts that can serve as the basis of an ensuing discussion.

In the process of recruiting students into the discussion, the teacher reports or clarifies student explanations as a means of emphasizing their mathematical qualities and bringing them to the attention of other students. By recruiting student explanations and skillfully using them as the basis of discussion, the teacher acknowledges the students as competent mathematical thinkers and establishes the community of students as the source of important mathematical ideas.

By "aligning students with positions in the collective argument," the teacher helps students notice the differences in the mathematical ideas embedded in student explanations. The teacher uses alignment to help students make connections, establish criteria for what counts as proper justification, and focus on particular concepts. As students see how their ideas contrast with other students' ways of thinking, they have the opportunity to reflect on their own solutions and begin the process of revision, often by incorporating other students' thinking. The result for the classroom community is often an explanation that is better than any one single student could produce.

The teacher's subtle actions to initiate a discussion include attributing "attitude and intention" in order to portray students as engaged and capable participants in the discussion. Linguists call this process *animation* because the teacher talks as if it were the students who are taking positions in a discussion even when the students have not yet grasped the significance of their claims. The teacher might use animation to juxtapose two students' explanations that differ in a mathematically important way.

Table 1

$y = 2^x$

x	y
1	2
2	4
3	8
4	16
5	32

Table 2

$y = 2^{x-1}$

x	y
1	1
2	2
3	4
4	8
5	16

A CLASSROOM EXAMPLE

Consider two students in an algebra class who are discussing the relationship between the two patterns in **tables 1** and **2**. The class has already determined that the equation to model the relationship given in **table 1** is $y = 2^x$ (where x is a natural number) and is trying to determine an equation for the relationship in **table 2**.

As the teacher circulates around the room during group work, she notices that Delilah has observed that the data in the y-column of **table 2** are half the corresponding values in the y-column of **table 1** and has written the new formula as $y = (1/2)(2^x)$. Similarly, the teacher notices that Kibwei has observed that the y-values of **table 2** are shifted down one space from the y-values in **table 1** and has written the new equation as $y = 2^{x-1}$. The teacher sees this as an opportunity to connect division with exponential notation. What follows is how the teacher orchestrates the discussion between Delilah and Kibwei.

Teacher. Delilah, what did you get for your equation?

Delilah. I noticed that each value was one-half of the old one, so I put a half times 2 to the x. [Teacher writes $y = (1/2)(2^x)$.]

Teacher. Can your answer also be written as 2 to the x divided by 2?

Delilah. Multiplying by a half is the same as dividing by 2. Okay. [Teacher writes

$$y = \frac{2^x}{2}.]$$

Teacher. Kibwei, what about you?

Kibwei. When I looked at the new table, I noticed that the values were moved one box down. I tried 2^{x+1} but that didn't work. Then I tried 2^{x-1} and it worked.

Teacher. Okay. So Delilah thinks you can divide by 2, and Kibwei thinks you subtract 1 from the exponent. Are Delilah and Kibwei disagreeing with each other, or is there a way to reconcile their answers?

Luis. Well, they both work, so I'm guessing they're both right, but I haven't figured out why.

Kibwei. It would have to be that dividing by 2 is the same as subtracting 1 from the exponent.

Delilah. Is it possible to write division with exponents, like we do with multiplication?

Teacher. I think we're close to figuring out an important relationship. Can somebody state what relationship we have just established here?

In this episode, the teacher recruits Delilah's and Kibwei's explanations after observing their responses during group work. The teacher elicits their explanations and then broadcasts them with subtle clarifications while explicitly attributing the ideas to the students. The teacher then positions Delilah and Kibwei as coparticipants in a discussion in which the differences between the two need to be resolved. The students are able to resolve these differences because they recognize that both expressions accurately represent the same data. The teacher concludes the discussion by seeking an articulation of the relationship described by the students. Here the teacher may want to talk about what happens when the exponent is 0, as in Kibwei's formula. This subtle yet important mathematical topic arises naturally through student discussion. Although most discussions do not work this efficiently, this episode provides an illustration of how a strong teacher presence can highlight important mathematics while helping students build on the ideas of others.

BENEFITS TO STUDENTS

Students benefit from participation in teacher-orchestrated classroom arguments in several ways. The first benefit is that they receive feedback on their way of thinking about a problem. The feedback allows students to see how their thinking correlates with that of other students in the class as well as with conventional mathematical ideas. It also gives students a chance to revise and refine their thinking from an initial "first draft" formulation. A second benefit is that students can influence how ideas are developed in the classroom community. Gee (2003) points out the importance of these two aspects of interactivity—feedback and impact—on the development of competency. His study of video-game players suggests that a student's ability to influence or manipulate an environment can affect his or her commitment to learning. The same applies to developing competency in mathematical discourse. A third benefit is that students learn to reference one another's ideas as they present their own explanations. Instead of communicating their solutions serially, students learn to play off one another's ideas and build a greater understanding of a problem or concept collectively. The practice of explicitly referencing and building on the ideas of others is a feature of academic and professional discourse; consequently, it is important to give students a chance to participate in mathematical argumentation. All these benefits together help students begin to develop identities as thinkers in a mathematical community and understand what it means to participate in mathematical discourse.

THE COMPLEXITY OF ENACTING ARGUMENTS

The demands imposed on teachers by teacher-orchestrated classroom discourse are considerably greater than for more traditional types of discourse, which typically take the form of recitation. In a traditional classroom discourse pattern, the teacher asks questions to which she already knows the answer and to which there is usually a single, correct, short-answer response. In teacher-orchestrated classroom argument, the teacher attends to multiple explanations and how those explanations relate to one another as well as to the underlying mathematical ideas. This approach magnifies the complexity of the teacher's real-time decision making: The interactions are less predictable, and teachers must take additional considerations into account as they react to an explanation.

The following brief classroom episode illustrates the complexity of establishing teacher-orchestrated classroom arguments. The students were asked to estimate the number of knee bends a person would do in 25 seconds if he had completed 18 knee bends in 20 seconds and 25 knee bends in 30 seconds. Although the data are not linear, the case could be made that in this short time period they are virtually linear. The teacher expected the students to use a linear interpolation strategy, which might lead to averaging the number of knee bends.

Teacher. Raymond, why did you say 23?

Raymond. I put 23 because if they added 5 seconds on to it, I think they would add 5 more [knee bends] to 18. 'Cause right here it says in 20 seconds he had 18.

Teacher. What do you think about that? Melinda?

Melinda. Raymond says there were 5 knee bends in [the first] 5 seconds, so the other 5 seconds he had 2 [knee bends]. I don't know. Shouldn't it be in the middle?

Consider what the teacher must think about in a very few seconds before continuing the discussion. Should she question Raymond's strategy? If Raymond continued his reasoning, would it not be natural to expect that at 30 seconds (5 seconds after the 25 seconds) there should be 5 more knee bends, thus giving us 28? Yet the data show only 25. Clearly, Raymond's reasoning does not fit the pattern in the data.

Also think about what Melinda has said and what is meant by "shouldn't it be in the middle?" Her question implies that the answer for 25 seconds should be somewhere between 18 and 23 knee bends and not simply the 23 knee bends that Raymond suggested. The only hint of how one might choose the correct value is the class's previous work in estimating values between two data points.

The teacher's reactions to these competing questions and explanations are not trivial but should take into account the following:

- The quality of the evidence presented for each explanation
- The differences between the explanations
- The mathematical ideas embedded in each individual explanation, such as rate of change (possibly but not necessarily)
- The mathematical strategy the teacher wants to emphasize (note that one strategy could be linear interpolation, but this may not be the best example for that strategy because the context is not linear)
- The teacher's knowledge of each individual's history in the classroom community

To illustrate the complexity of the teacher's decision making, I highlight one possible interpretation of the episode and show how this interpretation might frame the teacher's response.

Raymond's reasoning is ambiguous and could be interpreted as using either an additive or multiplicative strategy. He may have thought that adding five seconds would mean an additional five knee bends. Or he may have used a multiplicative strategy, in which the person was doing knee bends at a rate of roughly one knee bend per second for the first 20 seconds. However, the pattern implied by either strategy would not hold over the full 10 seconds delimited in the problem. Melinda responds by making two points. First, she notes the contradiction in Raymond's response. Then she hints at an alternative strategy, one that might be consistent with averaging.

The teacher must consider how to handle the differences between the two explanations and how to use these differences to underscore underlying mathematical ideas. The teacher could focus on the contradictions in Raymond's strategy as a means of suggesting that the rate of performing knee bends should remain fairly constant across the two intervals. She could simply build on Melinda's suggestion that the numbers be "in the middle" without reference to Raymond's claim. She could ask the students to elaborate on their explanations in order to determine the warrants for their claims. Or she could decide to open a class discussion and let other students compare the two explanations. The direction the teacher takes depends on pacing concerns, an evaluation of the importance of the mathematics that may emerge from discussion, and whether she has understood the nature of the explanations.

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Each action will influence ensuing responses, after which the teacher will need to make yet another decision about how to proceed in the discussion. The teacher must continually reflect on multiple considerations while involved in a dynamic set of interactions. This example illustrates the complexity of the decisions that teachers must make to orchestrate discussions.

DEVELOPING COMPETENCY IN ORCHESTRATING DISCUSSIONS

Teachers develop competency in orchestrating discussions in a way similar to how people develop expertise in any demanding profession: They learn through experience, collaboration, and reflection. Experience comes through practice and involves developing fluency with procedures and then with new ideas and techniques. Collaboration involves sustained mutual interactions through some shared practice with peers and those more expert. However, experience and collaboration would lead to little learning if people did not reflect on their practice and the content of the collaborative interactions.

What would this process mean for mathematics teachers interested in transforming their discourse practices? First, it means that teachers should become fluent in less demanding techniques before attempting new ones. For example, having students explain their reasoning is a good first step. After becoming competent in eliciting individual explanations and connecting those explanations to conventional mathematical ideas, teachers can then experiment with eliciting and coordinating multiple explanations. Second, this process means seeking a professional community whose members share a similar interest in transforming classroom discourse. Ideally, this community would include members well versed in the goals and principles of teacher-orchestrated classroom arguments. Third, this process means reflecting continually and systematically on the interactions and outcomes of attempts to orchestrate classroom arguments. This step may mean informal and spontaneous moments of reflection or a more formal attempt at action research.

These kinds of professional learning opportunities are not common. Teaching has historically been a rather isolated profession, a fact that has limited the kinds of learning environments it takes to transform core teaching practices. Good learning environments explicitly structure interactions to take advantage of the collective expertise of a community and maximize opportunities for collaboration. Although these learning environments are difficult to achieve, it is possible to create such opportunities by identifying colleagues or professional development projects that share an interest in transforming discourse practices.

CONCLUSION

Deep learners in any discipline must possess the ability to interact with ideas, offer evidence for one's propositions, and react thoughtfully and constructively to others' propositions. The skills that students develop in teacher-orchestrated classroom arguments reflect those characteristics of disciplinary learning and help them develop rigorous and sophisticated uses of mathematical language.

The strong teacher role articulated in this vision of discourse is at odds with some conceptions of student-centered discussion. However, the kinds of discourse practices advocated here and in the NCTM Standards (1989, 2000) represent a fundamental change from traditional practices for both teachers and students. Initially, at least, teachers need to provide explicit support and modeling to help students learn to reflect on and build on their peers' explanations.

Learning how to orchestrate classroom discussions looks much like learning in any demanding profession and, coincidentally, quite different from traditional professional development. To change teaching practices, teachers need to collaborate and reflect in the context of a professional community as they gain experience in the new practice.

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