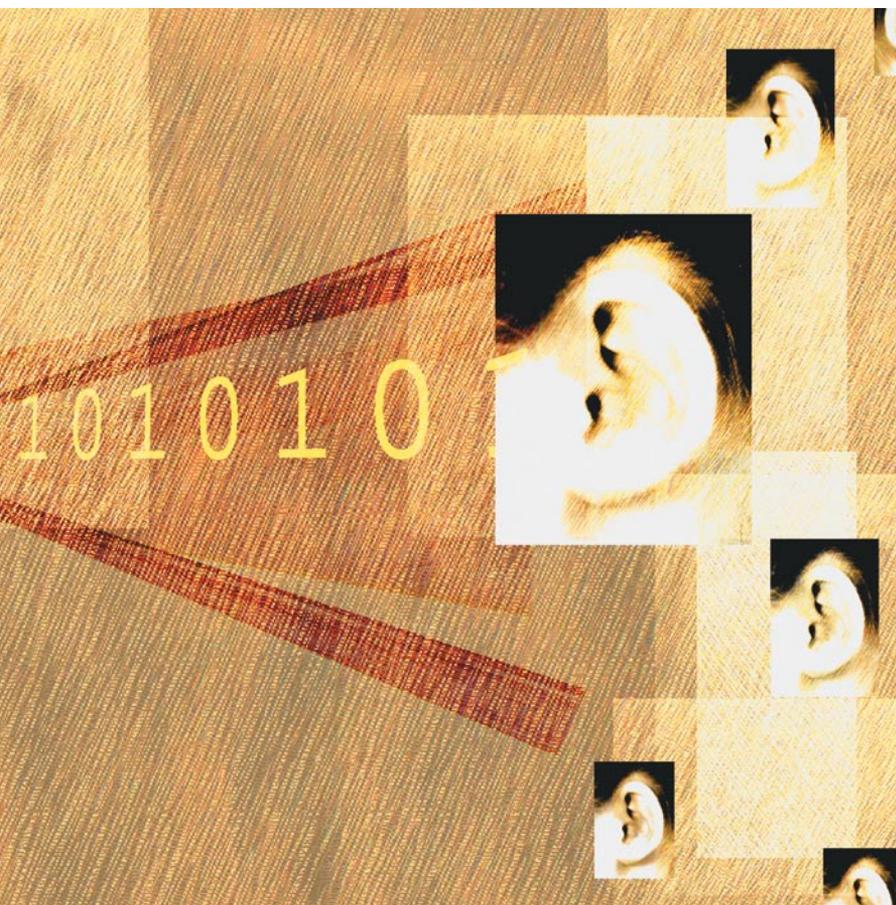


MATHEMATICS DISCOURSE

“It’s Like Hearing a Foreign Language”

Donna Kotsopoulos



“I know how to do it but the question is confusing. I don’t understand the words that the question asks.”

—Jacob, age 15 (throughout this article, pseudonyms are used to protect the identity of participants)

Jacob’s lament is all too familiar to teachers of mathematics. Too frequently I have heard students comment that hearing the language used in mathematics is “like hearing a foreign language.” This perception is an important consideration for mathematics educators in understanding the ways in which students experience learning difficulties.

To investigate this perception, I stepped outside my own classroom and studied another teacher’s ninth-grade mathematics classroom. I tape-recorded classroom discussions and then conducted interviews with the teacher and various students. This article explains what I saw and, most important, what I heard.

One conclusion from my study that might explain Jacob’s lament—the perception that the language of mathematics is like a foreign language—is that students experience interference when language is borrowed from their everyday lives and used in their mathematics world. When I use the word *language*, I am referring to the names given to mathematical objects (e.g., *table*, *factor*, etc.) and the ways we use

and work with these objects (e.g., *reduce*, *round*, *eliminate*, etc.) (Ben-Yehuda et al. 2005).

THE TABLE WITH FOUR LEGS OR THREE COLUMNS?

Some of the language used to talk about mathematics sounds familiar because it is familiar. Aspects of the language we use to talk about mathematics are borrowed from our everyday language (Dale and Cuevas 1992; NCTM 2000; Pimm 1987; Winslow 1998). This everyday language is colloquial, common, and familiar and includes conversational

language (Chamot and O'Malley 1994; Delpit 1998; Heath 1983; Orr 1987; Peregoy and Boyle 1997).

In contrast to everyday language, the mathematical register (Halliday 1978; Pimm 1987), which is unique to mathematics, is highly formalized and includes symbols, pictures, words, and numbers. Some of its aspects are unique to itself, whereas other aspects are borrowed from everyday language

and then used in unique ways (e.g., *table*). Because the mathematical register is used in unique ways (primarily limited to actions on or related to quantities of things), it is not easily usable outside the mathematics classroom, not even in other academic classrooms (Dahl 2004; NCTM 2000; Pimm 1987; Winslow 1998).

Words such as *cancel*, *if*, *limit*, and *table*, for example, must be relearned within the mathematical register. Other words, such as *parabola*, *quotient*, and *hypotenuse*, must be learned for the first time. Students must continually and actively negotiate among the mathematical meaning of a word, its everyday language meaning, and its new meaning as well as its alternative meanings within the mathematical register. Consequently, the mathematical register, unless made explicit to students, can indeed sound, feel, and look much like a foreign language.

A student's inability to successfully minimize interference can potentially undermine his or her ability to learn. Nesher, Hershkovitz, and Novotna (2003) show just this in their study of students engaging in mathematical problem solving: Students who are unable to negotiate discourse are unable to move forward in their learning. The multiplicity of representations of words in everyday language and within the mathematical register can create significant interference as students struggle to assign appropriate meanings to words in unfamiliar contexts. As a result, developing the math-

ematical register can be difficult for students unless similarities and differences are made explicit.

SEEING AND HEARING INTERFERENCE

To study language interference, I used transcriptions of classroom discussions and semistructured interviews with students. The examples that follow reflect, first, a teacher talking to the whole class and, second, interviews with students who were in the class.

Teacher-talk Interference

Teacher-talk interference results from the predominant use of the mathematical register by the teacher in the classroom. The teacher in my study spoke primarily in the mathematical register, even when introducing new concepts. I determined this by reviewing 300 minutes of classroom transcriptions, focusing on sixty words that I had identified as "belonging to the mathematical register." In the course of the 300 minutes, these sixty words were used more than a total of 1500 times. Take, for example, the following excerpt, in which the teacher is discussing orders of operations (note: the italicized words are identified as belonging to the mathematical register):

Teacher. This is our last topic in *algebra*, and it's actually not going to be terribly different from the stuff that you've already done. This is why it leads nicely into the review exercises, which prepare you nicely for the test. That is my plan: Do this topic, do the review exercises, and finish the morning with our review. *Adding and subtracting polynomials.* All right. Again, a lot of times I find people look at a question like that and they go home and say, "Look at all those *terms*, look at all those *positives* and *negatives*, look at all those *exponents*. I can't do that," and [they] throw up their hands in frustration. But all it would take is for them to take two seconds and look at it and realize, "Wait a minute, what's the *operation* that I'm being asked to *perform* here? What's the *operation* I'm being asked to *perform*? And how can I rely on prior knowledge?" Watch. What's the *operation* here? [*long pause; the teacher calls on a student whose hand is up*]

Evan. Division. Brackets, basically multiplication.

Teacher. I don't think so.

Evan. Addition? What was the question again?

Teacher. You've got four choices. What is the *operation* here?

Evan. [*shrugs*]

Teacher. I don't think so. To look at it, you've got a *set of brackets*, but the important part is what's in between them. It's the *positive*, so you're being asked to *add* this *polynomial*. What kind of *polynomial* is this? A *trinomial*. With this *trinomial* right here. All right. Now before we

Students experience interference when language is borrowed from their everyday lives and used in their mathematics world

do *BEDMAS* [an acronym referring to the order of operations (brackets, exponents, division, multiplication, addition, and subtraction)], . . . according to *BEDMAS*, we have to *do brackets* first, right? And when we say *bracket*, we mean *everything inside* the brackets. Well, can you *do anything* with what's in the *brackets* right there? Are we *done*?

Of importance is the teacher's wide use of the mathematical register in the absence of any qualification or elaboration. The teacher uses the words *do anything*—this phrase too is transformed within the mathematical register, indicating that the act of doing is distinctly different in mathematics. What does *doing* mean within the mathematical register? As we saw, the student, Evan, was unable to respond to the question.

To determine whether students followed along in this use of the mathematical register during this lesson, I asked them during the interviews to explain to me what a *polynomial* was. Only one student interviewed put forth an adequate, although vague, explanation. When I asked the students to show me a polynomial in their notebook, several did so; however, they prefaced their selection by “I think” or “I'm not sure” or “Is that correct?” The false assumption that students were familiar with the mathematical register as well as the teacher's casual use of the mathematical interpretations of everyday language and technical mathematical terms interfered with the students' understanding (Bernstein 1972; Blom and Gumperz 1972; Gumperz 1982; Orr 1987; Pimm 1987).

In the previous excerpt and many other examples of classroom discussions, the teacher-talk, primarily in the mathematical register, dominated classroom discussions. Teacher-talk represented upward of 80 percent of the transcribed data. The teacher dominated the classroom discourse, preventing students from actively participating (Gustafson and MacDonald 2004; Hiebert et al. 2004). We see that the student was unable to attend to the question asked.

Student-talk Interference

Student-talk interference occurs when students talk about mathematics with one another using everyday language. When students talk with one another, they are most likely to speak in peer-appropriate everyday language (Cummins 1984; Pimm 1987). Consequently, they fail to develop their understanding of the mathematical register or, worse, develop misconceptions, as the next excerpt illustrates.

To explore the extent to which interference occurred in student-talk, students were asked to explain a problem as if they were explaining it to a peer. The problem involved the multiplication of

one monomial by another and had an error in the solution: $4x(3x^3) = 12x^3$. If the students understood that the problem was solved incorrectly, they were then asked to explain how to solve it correctly. Following is an excerpt from Joanna's interview:

Joanna. Okay, that would be like $4x$ times $3x$ and I know how to do that, I did that in class. Okay, the first thing that I would do would be . . . [*long pause*].

Interviewer. Show me. Use the pencil and it might be easier.

Joanna. It would be $4x$ times $3x$ twice, because it has a . . . what do you call that? I don't know. I *know how to do it!* [*Joanna's emphasis*]

Interviewer. Write it out, show me.

Joanna. Okay, it would be $4x$ times $3x^2$, okay, but then you write it twice because there's a 2 up there, right, so you go once, $3x$, and you do it twice because there's a 2 up there, and so you're done with this, and then you times this and you go here, you go, what's 2 times . . .

Interviewer. What is that method called?

Joanna. Rainbow thing.

To a person unfamiliar with mathematics—another student, for example—it might appear that Joanna, because of the authority with which she completed the task, actually understood the mathematics. A closer examination of her explanation, however, revealed that she has a limited understanding of how to multiply two monomials. She was able to retrieve the word *rainbow* to describe the distributive property; however, this metaphorical code-switching (Blom and Gumperz 1972) did not enable her to engage in an appropriate strategy to move herself forward in her problem solving.

Particularly problematic is her understanding of exponents. This example shows that Joanna has minimal language proficiency in the mathematical register (both orally and symbolically), which prevented her from being able to explain the problem.

Students struggle with the distinctions between everyday language and the mathematical register. In the course of the interviews, I asked students to explain words from the mathematical register; they could use pictures, numbers, or words in their explanations. Regardless, all students gave oral explanations. The following excerpt highlights interference from everyday language, as experienced by Ryan:

Despite my heightened awareness . . . I found that the mathematical register continued to dominate even my own teacher voice

Interviewer. Your advice was to “*expand it*”—what did you mean by this?

Ryan. Expand? Expand, expand, what does it mean? I don’t know. Um, not really, expand. Expand, make it bigger, stretch it, expand it. That’s probably what it means in math.

Interviewer. What did you mean by *simplify*?

Ryan. Simplify in math, like find the answer and work it out.

Interviewer. Now, what’s the difference between *simplify* and *evaluate*?

Ryan. Evaluate is where, evaluate would be evaluate.

Interviewer. Do you get an exact answer with *simplify* and *evaluate*?

Ryan. Not really.

Interviewer. How are they different? Can you show me an example from your work?

Ryan. I don’t know. I know [the] difference, but . . . No, I don’t know. Probably something is wrong or something, and you never know.

Ryan was asked to *expand*, which means to multiply everything inside the brackets by the term outside the brackets—that is, to apply the distributive property. Examination of this response and the other interview data shows that students often use everyday language inappropriately rather than using the language of the mathematical register. When students have gaps in understanding and when imprecise language is used, the interference may be insurmountable.

INTERFERENCE FROM WITHIN THE MATHEMATICAL REGISTER

One other type of interference that I observed in my study arises from students’ inability to distinguish between alternative meanings of terms from within the mathematical register. For example, in what instances will one solution be correct, as opposed to none, two, or more than two? I call this textual interference.

Textual interference arises when students are not able to discern the appropriate use of a particular word or term from the mathematical register because they are not able to make sense of the mathematical context. This type of interference could be related to interference from everyday language, but this may not necessarily be the case. For example, a question asking a student to *factor* may elicit a number of possible responses:

- (a) 24: 8 factors
- (b) $x^2 + x - 2 = (x - 1)(x + 2)$: 2 factors
- (c) $3x^2 + x + 9$: no factors other than itself and 1

Another example can be seen in the inappropriate but common use of the word *cancel*:

$$(a) \frac{(x-\cancel{3})(x+2)}{(x-\cancel{3})(x+1)}$$

$$(b) \cancel{x} - \cancel{x} = 0$$

In example (a), *canceling* results in 1, whereas in example (b) *canceling* results in 0. Neither use of *canceling* is mathematically accurate, further exemplifying why students experience challenges. Cummins (1984) says that meaning and language can be developed simultaneously through contextualized problems that require students to talk about what they are learning as they are learning it. At first, the language may not be precise, but as students continue to work together and talk with one another and the teacher, the underlying meanings of the words evolve.

INTERFERING WITH INTERFERENCE

Following this research, I returned to my own ninth-grade classroom and taped my own classroom discourse. Despite my heightened awareness of how students and teachers use language differently, I found that the mathematical register continued to dominate even my own teacher voice. I was equally surprised at my dominance of classroom discourse overall. This realization showed me that heightened awareness is not enough, and that is why the November 2007 focus issue of *Mathematic Teacher* is so important.

Classrooms are diverse social settings. As mathematics educators, we need to know how we use language to build meaning in mathematics and be aware of how our use of the mathematical register may limit our students’ participation. This awareness requires that we reconceptualize the teaching and learning of mathematics. Some important work has already been done in this area around code-switching—that is, the use of everyday language to build the mathematical register (Adler 1998; Zazkis 2000).

We need to be explicit and intentional in developing the mathematical register. I am not advocating a return to a prehistoric learning-by-definition model of mathematics education. As Sfard et al. (1998) point out, definitions are not a shortcut to meaning in mathematics. I am suggesting that it is important to set up learning opportunities for students to use mathematical language themselves to be able to see through the outwardly familiar language to the underlying mathematical meaning (Adler 1999).

Although the discourse in a mathematical classroom is seemingly familiar, largely because of the extensive use of everyday language, it is nevertheless new for the student in both meaning and use. I challenge all of us, as teachers, to consider thoughtfully the discourse that occurs in our classroom and in what contexts students experience the discourse. The teacher in this research talked approximately 80 percent of the time, a percentage, research shows, that is not uncommon in mathe-

mathematics classrooms (Hiebert et al. 2004). Students talked only 20 percent of the time, and some students did not talk at all. To become proficient in mathematics, students need to participate in mathematical discussions and conversations in classrooms. This participation, in turn, will allow teachers to understand better whether students are making appropriate conceptual connections between words and their mathematical meanings. I also encourage teachers to tape-record themselves and reflect on what they hear. Tape-recording will, unfortunately, make clear why students think mathematics sounds like a foreign language.

REFERENCES

- Adler, J. "A Language of Teaching Dilemmas: Unlocking the Complex Multilingual Secondary Mathematics Classroom." *For the Learning of Mathematics* 18, no. 1 (1998): 24–33.
- . "The Dilemma of Transparency: Seeing and Seeing through Talk in the Mathematics Classroom." *Journal for Research in Mathematics Education* 30, no. 1 (1999): 47–64.
- Ben-Yehuda, M., I. Lavy, L. Linchevski, and A. Sfard. "Doing Wrong with Words: What Bars Students' Access to Arithmetical Discourses." *Journal for Research in Mathematics Education* 36, no. 3 (2005): 176–247.
- Bernstein, B. "A Sociolinguistic Approach to Socialization; with Some Reference to Educability." In *Directions in Sociolinguistics*, edited by J. J. Gumperz and D. H. Hymes, pp. 465–97. New York: Holt, Rinehart and Winston, 1972.
- Blom, J., and J. J. Gumperz. "Social Meaning in Linguistic Structure: Code-switching in Norway." In *Directions in Sociolinguistics*, edited by J. J. Gumperz and D. H. Hymes, pp. 407–34. New York: Holt, Rinehart and Winston, 1972.
- Chamot, A., and J. M. O'Malley. *The CALLA Handbook: Implementing the Cognitive Academic Language Learning Approach*. Boston: Addison-Wesley Publishing Co., 1994.
- Cummins, J. *Bilingualism and Special Education: Issues in Assessment and Pedagogy*. Clevedon, England: Multilingual Matters, 1984.
- Dahl, B. "Analysing Cognitive Learning Processes through Group Interviews of Successful High School Pupils: Development and Use of a Model." *Educational Studies in Mathematics* 56 (2004): 129–55.
- Dale, T. C., and G. J. Cuevas. "Integrated Mathematics and Language Learning." In *The Multicultural Classroom: Readings for Content-Area Teachers*, edited by P. A. Richard-Amato and M. A. Snow. White Plains, NY: Longman, 1992.
- Delpit, L. D. "Ebonics and Culturally Responsive Instruction." In *The Real Ebonics Debate*, edited by T. Perry and L. D. Delpit, pp. 17–26. Milwaukee, WI: Rethinking Schools, 1998.
- Gumperz, J. J. *Discourse Strategies*. Cambridge: Cambridge University Press, 1982.
- Gustafson, B. J., and D. MacDonald. "Talk as a Tool for Thinking: Using Professional Discourse Practices to Frame Children's Design Technology Talk." *Canadian Journal of Science, Mathematics and Technology Education* 4, no. 3 (2004): 331–51.
- Halliday, M. A. K. "Sociolinguistic Aspects of Mathematical Education." In *Language as Social Semiotic: The Social Interpretation of Language and Meaning*, pp. 194–204. London: Edward Arnold, 1978.
- Heath, S. B. *Ways with Words: Language, Life, and Work in Communities and Classrooms*. Cambridge: Cambridge University Press, 1983.
- Hiebert, J., R. Gallimore, H. Garneir, K. Bogard Givvin, H. Hollingsworth, J. Jacobs, et al. *Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study (NCES 2003–13 Revised)*. Washington, DC: U.S. Department of Education, National Center for Education Statistics, 2004.
- National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.
- Nesher, P., S. Hershkovitz, and J. Novotna. "Situation Model, Text Base and What Else? Factors Affecting Problem Solving." *Educational Studies in Mathematics* 52 (2003): 151–76.
- Orr, E. W. *Twice as Less: Black English and the Performance of Black Students in Mathematics and Science*. New York: Norton, 1987.
- Peregoy, S. F., and O. F. Boyle. *Reading, Writing, and Learning in ESL*. New York: Longman Publishers, 1997.
- Pimm, D. *Speaking Mathematically: Communication in Mathematics Classrooms*. New York: Routledge and Kegan Paul, 1987.
- Sfard, A., P. Nesher, L. Streefland, P. Cobb, and J. Mason. "Learning Mathematics through Conversation: Is It as Good as They Say? [1]." *For the Learning of Mathematics* 18, no. 1 (1998): 41–51.
- Winslow, C. "A Linguistic Approach to the Justification Problem in Mathematics Education." *For the Learning of Mathematics* 18, no. 1 (1998): 17–23.
- Zazkis, R. "Using Code-switching as a Tool for Learning Mathematical Language." *For the Learning of Mathematics* 20, no. 3 (2000): 12–22. ∞

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