Making the Most of Mathematical Discussions

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The importance of mathematical discourse and its connection to developing conceptual understanding, communication, and reasoning is well documented throughout NCTM's *Principles and Standards for School Mathematics* (2000). For example, NCTM's Learning Principle emphasizes the role of discourse in supporting student learning, noting that "classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge (Lampert 1986)" (NCTM 2000, p. 21). The skillful facilitation of discussions is something both novice and experienced teachers find challenging. Most teachers can recall a well-planned lesson that did not unfold as expected. From this article, we hope readers gain insight into planning mathematically focused, collaborative discussions. We illuminate three key aspects of the pedagogy of teachers who were successful in consistently organizing whole-class discussions. These teachers created learning environments aligned with NCTM's vision of good practice, where students were given conceptually demanding tasks, worked together to develop ideas, and consistently were asked to make sense of mathematics.

We focus on two kinds of classroom discussions: *sharing discussions* and *collaborative discussions.* In sharing discussions, students share their answers and the teacher values their contributions. Students are respectful of one another's ideas and may think about others' ideas in relation to their own; however, they often maintain a primary focus on their own reasoning process. In collaborative discussions, students also share their ideas, but, in addition, they build on the thinking of their classmates' responses, take up classmates' ideas, and work with these ideas explicitly to extend the line of thinking that emanates. This approach pushes students to develop new understandings of mathematics. We capture differences in these kinds of discussions by talking about three key aspects of pedagogy: positioning students for discussion, managing wrong answers, and connecting and linking across ideas. These key aspects were derived from observations and analyses of videotaped lessons of three teachers as well as from our own teaching experiences.

POSITIONING STUDENTS TO ATTEND TO ONE ANOTHER'S IDEAS

A teacher's comments can direct students' thinking and indicate what he or she expects to happen next. For sharing discussions, one format is to have several students (or groups) explain their ideas. The teacher can follow up a student's explanation with "Does anyone have another idea?" or "Let's see what group 5 did." These comments focus students' attention on the array of different ways to think about a problem and indicate that students are expected to attend to others' lines of reasoning. The teacher often checks in to be sure students understand others' ideas and methods by asking, for example, "Does anyone have a question about Sue's method?" In a sharing discussion, students are expected to understand what others do, but they maintain a primary connection to their own ideas. This format can support many learning goals, including communication and reasoning.

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In collaborative discussions, the teacher's comments direct students not only to understand what others are saying but also to respond to, extend, or connect to those ideas. For example, the teacher can ask, "Does anyone want to build on what Sally just said?" Sometimes these kinds of exchanges happen without the teacher's active positioning of students to respond to one another. However, the teacher can encourage students to work directly with one another's ideas and can take steps to foster such interactions. The following example, drawn from a ninth-grade classroom of lower-attaining students, demonstrates this. The task was for groups to generate sets of three ages that added to 90. These ages had to be of three different generations, so there needed to be at least 15 years between each pair of ages. One student, Frank, wrote his group's first set of ages—15, 30, and 45—on the board and then added a second set—15, 32, and 43. Without commenting, the teacher asked Frank to explain his thinking. The class likely could not hear his quiet response, so the teacher asked him to repeat it:

Teacher. Would you say that louder, because that's a very nice strategy, Frank. Would you stand to this side, please?
Frank. We just added 2 from that one [*points to the 45*] and then subtract, subtract, er, yeah, we just added 2 to that one [*points to the 32*] and then subtracted 2 from that one [*points to the 43*].

The teacher repeated the strategy and asked for questions. Ron raised his hand and tried to get Frank's attention.

Teacher. Ron, did you have a question for Frank?
Ron. Yeah, 32 and 43 are too close.

The teacher thanked Ron, had another student explain what Ron meant, and then asked the class to "use Frank's strategy" to generate a valid set of ages. This response encouraged the class to take up Frank's general logic and work the problem accordingly, keeping in mind the constraint that the values had to be at least 15 apart. This active positioning of students—in which the teacher asked Ron if he had a question for Frank, asked another student to explain Ron's comment, and then directed students to "use Frank's strategy"—makes students more likely to take up one another's ideas and thus fosters collaboration. It also helps develop norms of listening to and thinking about other people's ways of working, which further supports collaborative discussions.

In collaborative discussions, although a student's original idea serves as a reference point for her own work on the problem, students are asked to connect their ideas to others' or move away from their original ideas and consider the problem from a new perspective. In the episode above, all students had solved the problem in some way, but they were now being asked to think about Frank's strategy, which allowed the teacher to focus on ideas of equivalence and begin a conversation around that new idea.

To get students to listen to, comprehend, and extend others' ideas, teachers must ask them to do this consistently and not only in a whole-class format. For example, during group work, the teacher can actively support collaborative discussions by engaging all students at the table in the conversation. In one classroom, the teacher responded only to “group questions”—questions that no one in the group could answer. This approach implicitly positions students to consider one another’s questions, responses, and ways of reasoning and promotes dialogue among group members that can turn into collaborative mathematical discussions.

MANAGING “WRONG” ANSWERS

Many teachers have taken steps to reduce the stigma attached to being wrong, thus communicating to students that mistakes in the learning process are acceptable. The students in these classrooms were comfortable sharing incorrect answers because the teachers had established a classroom environment that accepted “wrong” answers. We noticed differences, however, in how “wrong” answers were managed across different kinds of discussions, which in turn promoted different kinds of conversations.

In sharing discussions, when a student offered a mathematically incorrect or incomplete idea, the teacher politely noted it as incorrect or elicited additional ideas, until one or more ideas that the teacher could productively work with and build on emerged. This approach can be a productive and appropriate strategy, particularly if the generation of multiple ideas is then used to support student analysis. However, if only the “correct” ideas regularly receive attention, the mathematics that gets explored is limited, and the students whose original ideas were incorrect may hold on to incorrect mathematics.

The following dialogue, taken from a geometry unit, highlights the sharing discussion. The teacher begins by asking what *area* means:

Teacher. What is area? Let’s start with the first one. What’s area?
Bill. Um, I had it, um—the amount of space an object takes up.
Greg. That’s volume.
Bill. Um, I had it for, like, a second.
Teacher. Okay. Bill said the amount of space an object takes up, and remember, we’re talking about area, so does anyone want to go off of that? Okay, Sue?
Sue. Okay, it’s like when you have any kind of a polygon, the area is the inside of that shape, in whatever measurement, squared, usually measurement squared.
Teacher. The inside, the space inside?
Sue. The space inside a polygon.

In this example, the first student is incorrect and is corrected by a classmate. The teacher acknowledges this wrong answer, but there seem to be no negative feelings or consequences associated with being incorrect.

In collaborative discussions, we observed that incorrect answers can be used to generate discussion. Focusing on a point of inconsistency or error has great potential for prompting new discussions. The episode involving Frank is an example of how an “error” was used to organize and continue student discussion of a problem. Interestingly, in collaborative discussions, we found it rare for something to be explicitly identified as “wrong.” Rather, students’ ideas were treated as “works in progress.” The focus of the teacher’s guidance was not to help students “correct” something that was wrong. Rather, it was to help the student and the class extend the idea that had been presented and continue to develop a viable solution collaboratively. One teacher, after students had worked for a while in groups, framed the class’s problem solving in the following way:

Teacher. All right, ladies and gentlemen. I am seeing so many neat ideas, and a lot of these ideas are contradicting one another. So we have got to figure out what makes sense and what doesn’t.

This comment set the stage for the subsequent discussion. It indicated that the class was to continue its exploration and that the upcoming presentations would represent “works in progress,” as opposed to indicating that some groups had the problem right and would show others what they had done.

CONNECTING AND LINKING ACROSS STUDENTS’ IDEAS

Consistent with NCTM’s Standards, many teachers have students share their ideas by presenting their work to classmates. This approach allows students in each group to use mathematical language and express their procedures and strategies to classmates. It also exposes other students to multiple approaches to problem solving, thus characterizing the sharing discussion.

We observed collaborative discussions that began with groups presenting their ideas but that evolved into conversations about new ideas and extended students’ prior thinking. During collaborative discussion, teachers connected student work to past classroom work and to other students’ work. For example, one teacher gave a problem showing two ways to find the area of a parallelogram. The groups were to determine which solution or solutions were correct and show how they knew. At the end of the allotted time, the teacher specifically asked three groups to present their solutions. She was deliberate in her selection and ordering of these presentations. The students in the first group explained how they used substitution of sets of numbers in both methods; the students in the second group showed how they used geometry to verify the approaches; and the students in the last group used algebra to "prove" that the two methods were the same. This sequencing allowed the teacher to open up a conversation about mathematical standards of rigor and what counts as a proof. Because she had checked in with the groups as they worked, the teacher could select and sequence the order of group presentations to enhance the classroom discussion. She targeted a particular mathematical goal and linked the student presentations with an eye to this goal. Although many discussions involve students making public their strategies and results, collaborative discussions bring to the fore the connectedness among student methods and ideas.

IDEAS FOR IMPLEMENTATION

Developing an extensive teaching repertoire, which includes both sharing and collaborative discussions, is a challenging and worthwhile goal. In this section, we suggest both broad and subtle changes that may produce greater student collaboration. Our challenge to the reader is to examine closely current teaching practices and experiment with the suggested strategies to promote collaborative discussions. Because collaborative discussions are more challenging to organize, we focus on implementation tips to enact this complex pedagogy.

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Positioning Students to Attend to One Another's Ideas

Collaborative discussions, to reiterate, link student ideas. To accomplish this linkage in your classroom, guide consecutive student responses to maintain focus on the same general idea. Specifically, when a student raises her hand, instead of just indicating her turn, ask whether she has a comment for the previous speaker or whether she can build on what was just said. If a student makes a comment without linking, ask a follow-up question, such as, "Is that different from what Ira said?" or "How is that related to Nate's comment?" Similarly, after a group presents, instead of asking, "Any questions?" change the phrasing slightly to "Ask the group a question" and pause for students to take up the invitation. The simple phrase indicates to students that they are expected to listen to and be prepared to interact with the speaker about the mathematics. It also reinforces the idea that the speaker's audience is his classmates and not just the teacher. This level of comfort develops over time, as students gain experience working in these ways.

During group work time, the teacher can reinforce linkage by assisting students in attending to others' ideas. If a student has his hand raised, walk toward the group and ask the group to listen to the question. Facilitate the group's discussion of the question or guide the original student in posing the question to the group.

Managing "Wrong" Answers

One approach is to think developmentally about your students. Your goal is to decide what they do not yet understand and help them move toward this understanding. Let a "wrong" answer be the catalyst for the class discussion and guide the students to identify what is correct about the response (see also Breyfogle and Herbel-Eisenmann 2004). For example, suppose students are trying to determine the cost of 5 gallons of gas at $2.19 per gallon. Darren replies, "$2.20 times 5 is $11.00, so I have to subtract 1 cent because $2.19 is 1 cent less than $2.20. So my answer is $10.99." After asking other students to consider Darren's response ("What do we think about Darren's answer?"), follow up by encouraging students to identify what Darren's misunderstanding might be. At the end of the discussion, highlight what students now understand that they did not before. Finally, thank Darren for offering the original idea and the opportunity to explore this piece of mathematics further.

Connecting and Linking across Students' Ideas

As you plan for a lesson or discussion, take notes on the ideas you expect students to generate. Consider what ideas might be linked productively and how the students' different responses can help you focus on important mathematical ideas. NCTM's Standards are a good resource for determining mathematical concepts experienced in previous grades. Consider the multiple access points of your students. What are the levels of understanding that students may bring to the lesson? During the activity, visit with or eavesdrop on groups to understand each group's process. In deciding on the order of presentations, consider where mathematical links can be made that are consistent with your planning. It is important, however, to be flexible enough to adapt to the flow of class discussion. Often, students bring up relevant mathematics that you had not planned for. And remember, not all ideas necessarily have to be shared each time. Most important are the connections and links.

We hope this article helps readers think about two types of classroom discussions—sharing and collaborative. Many components of each type may be familiar and implemented regularly. However, we suggest that considering three aspects of pedagogy—positioning of students for discussion, managing wrong answers, and connecting and linking across ideas as well as attending to how each may affect discourse in the classroom—can help elicit more mathematically powerful classroom discussions. As with any new implementation, time and consistency are required until students realize that they are now expected to respond and listen not only to the teacher but also to other students.

REFERENCES

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