A Writing Workshop in Mathematics

Community Practice of Content Discourse

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It was once again time to model a writing workshop in my graduate class for teachers and teacher candidates. The language arts and social studies majors were poised and ready, while the mathematics majors were asking if this exercise had any relevancy to their teaching. As a mathematician who seemed to be speaking for them had written, "One reason I chose mathematics for my undergraduate major was that it didn't require papers. Math homework called for solving problems or proving theorems, and that was just fine with me" (Burns 2004, p. 30). Many mathematicians in my classes believe that they are not qualified to "teach writing," reporting that their only pedagogical training in this area was their own experience in writing papers. Ause (1993) claims, "Our experiences as students and training as teachers often teach us two things about student writing: students write it and teachers read it—both, usually, in isolation. Writing seems to be full of silent suffering on everybody's part" (p. 162). A writing workshop can change all of this. My mathematics students have agreed that workshops are useful for dispelling writing fears, furthering understanding of mathematical processes, advancing student learning in writing strategies used by mathematicians, and expanding mathematical discourse through targeted discussion and cooperative work.

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For the purposes of this article, a writing workshop is defined as a structured peer collaboration that engages participants in thoughtful, controlled discussion of written assignments. Working in cooperative groups of approximately four, students respond to instructor-designed prompts to elicit feedback related to a particular project's standards. The process advances both oral and written forms of discipline-related discourse by facilitating interaction and language practice among students. Its cooperative nature is especially supportive of those who may have linguistic, cultural, ethnic, and skill-level differences.

WORKSHOP PROCESS

Writing workshops can be conducted in different ways, but some preliminary steps are typically taken before the actual workshop begins. Workshops emphasize the process of writing, as opposed to a final product, so, as a first step, it is useful to brainstorm for, or contemplate, the ways one might approach an assignment. At the secondary level, this can be done as a class (with the teacher as leader), in pairs or small groups, or alone. Next, students draft their ideas, attempting to meet the requirements of the assignment. These requirements should be carefully presented by the teacher in rubric form and shared with students before they begin writing. Once the paper is drafted, a rubric allows for self-evaluation that can be rehearsed before the actual peer workshop begins.

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Collecting drafts a day or two before the workshop ensures that students have done the preliminary work needed to participate. The main activity begins in the small collaborative group with members reading aloud shorter drafts or passing around longer drafts for response, a process that helps set the stage for writers to understand how other readers are "hearing" their work. The heart of a beneficial workshop is the peer feedback given by a relatively well-informed audience regarding the efficacy of the writing. Reader response should not be confused with "peer editing," which involves having students "correct" others' mathematical and grammatical errors, a process not recommended at the elementary school level—that will be the teacher's job down the line. Instead, respondents are helpful listeners, coaches, or sounding boards, sharing thoughts or ideas about the writer's attempts. Listeners, rather than trying to correct, will focus on issues such as these: Did the writer accomplish the task? What steps did the author use to solve the problem? Where did logic or reasoning break down? What was helpful, unique, or creative about the author's mathematical approach?

Being an astute listener and meticulous note taker is important for respondents and reinforces their own learning as they struggle to analyze and evaluate their peers' language and approaches to problem solving. There are many different ways to write, so remind students that effective writing comes in different forms. A strong technical writer may not win a poetry award. A science lab report does not sound like a literary essay. Dickens did not write like Hemingway, but both wrote highly praised novels. Some people just need the time to practice different mathematical genres. Let students know that workshops are safe places to do that work. Professional writers revise their work many times to improve their writing, but students composing papers are expected to "get it right" on the first draft. Workshops create an environment to "hear ourselves" as others hear us, try out the strategies and vocabulary of the discipline, and make revisions when necessary. Authorizing time to make revisions after discussion with peers allows students another chance to get things right.

If possible, before the class arrives, organize the room for teams of four students each. Within the teams, arrange desks tightly so that all teammates can see one another clearly and have a more private, comfortable space to work. However, separate groups from one another as much as possible to allow for a quieter classroom during discussion. Depending on the teacher's goal, groups can be differentiated heterogeneously or homogeneously, and this differentiation can be fluid, depending on the project and the teacher's goals. Planning in advance which students will work best together is an important step. Also, posting important rules on chart paper or using an overhead projector or PowerPoint can save time. This way the rules are available for quick review at each workshop.

At this point, let me introduce Laura. She was a high school student who had to be escorted through the halls to get to class from the "alternative" wing, where she spent most of the day. Something about a quick temper, too many fights, and probation factored in. Her social skills were "underdeveloped," but she was bright and verbal, though her choice of verbs was not always welcome in a school environment. Intimidated about participating in the "read-around portion" of the workshop, in which students share written work by reading aloud or pass papers around for their teammates to read, Laura had assumed judgment—which was more typical of her school interactions—rather than cooperation. She was pulled between the familiarity of a defiant "I won't do that" and the clear desire to demonstrate her own knowledge and function as part of a team.

Small-group interaction, which facilitates a cooperative atmosphere, can help students like Laura demonstrate critical mathematical skills without falling back on old behavior patterns or acting out for the larger audience. Her off-target behaviors decreased when she discovered how her natural openness, directness, and aptitude could help her succeed academically in the workshop. Her teammates' appreciation of her efforts was inordinately helpful in getting Laura to develop better social as well as mathematical strategies and kept her functioning in this one mainstream classroom long after she had failed in others. Writing workshops are about helping students figure out how to communicate effectively and giving them the language that fosters success. The workshop's scripted environment and clear guidelines gave Laura assistance in the discourse of both mathematical and social competence.

WHAT STUDENTS SAY AND DO IN WORKSHOPS

Assignments of three pages or fewer can be read aloud in the small group. Select which student will read first. Start with "the oldest," "the tallest," or, for example, "the one wearing the most blue"; even this basic criterion can help nervous participants begin comfortably. This first reading is followed by a round robin of response, beginning with the student to the reader's right and the others following in order. The writer should listen politely to all responses, take notes as needed, and say, "Thank you." Assigning respondents specific roles is especially useful when participants are new to workshops. It helps younger and less mature students stay on track and focused. Advanced students or those more experienced in workshop procedures may choose to ask listeners for specific feedback in areas they found difficult.

Examples of questions about their writing that mature students could ask respondents include the following. (It is best to avoid asking yes or no questions, but shaping good questions takes practice.)

- Where did you get confused in the piece, or what parts were hardest to understand?
- What was my strongest (clearest, most concise) point?
- Point out a place that seems illogical or incorrect or where reasoning breaks down.
- What part of my explanation most effectively demonstrated my understanding of the problem?
- Where was my mathematical terminology, representation, figures, or notation most useful or least effective for you as a listener?

Sometimes, giving all participants the assessment rubric for the assignment and having team members respond to different aspects works well. If you have a particularly immature group, you may want to tell students that response sheets will be collected at the end of class so that you can check their work. After the first or second workshop, this instruction is seldom necessary.

Remind writers to avoid defending or arguing about their work with respondents. The writer's job is to listen carefully, remembering that some students are better listeners or better analysts than others. Writers simply take in the information, thank the person, and later decide if the feedback should instigate change in the writing. The element of reflection is important for mathematical writers to practice.

I disallow the comments "That was really good" or "That was bad" as responses because they convey no useable information to the writer. Very specific responses—"The language used to describe the steps in that equation was clear and concise" or "The reasoning for moving from step 3 to step 4 is unclear. Could you justify your thinking?"—are more useful and teach students about how language and thinking work in mathematics. Coaching students to respond with specific information keeps the process more professional. Also, when teachers share a clear, concise rubric that details required standards using mathematical discourse, it gives respondents guidelines for formulating and articulating useful feedback.

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Some teachers like to hand out or post in the classroom a list of response phrases that encompass mathematical concepts. These help guide students in the ways language works, and teams enjoy adding to this response vocabulary as they become more adept. Examples of these response phrases include the following: "clear sequential path," "logical details," "concise steps," "fluid argument," "concrete example," "helpful analogy," "beneficial strategy," "creative methodology," "convoluted reasoning," "faulty logic," "addresses all components of task," and "effective procedure."

Specific categories of responses that have been

Introduction

1. The introduction restates the problem accurately.
2. It communicates the topic and signals a direction for the paper.
3. It is informative, clear, and concise.

Mathematical Process

1. Mathematical laws and properties are correctly stated.
2. The author-mathematician uses graphs, charts, formulas, and so forth to assist reader's understanding when needed.
3. The solution demonstrates adequate (or inadequate) understanding of the problem.
4. The author uses insightful and efficient (creative, complex, sophisticated) strategies.
5. The mathematical process gives specific detail as well as thorough explanation of necessary steps.
6. [For advanced students] The author provides more than one way to solve the problem.
7. [For advanced students] The author recognizes relevant patterns and generalizes to other problems.

Mathematical Discourse

1. Mathematical discourse is comprehensive.
2. All terminology used is accurate.
3. Language is (complex, advanced, clear, confusing, inaccurate).

Organization

1. The paper is well organized and guides the reader through clear, understandable steps.
2. Transition (signal) words ("first," "second," "next," "a final step") guide the reader.
3. Clear sections (beginning, middle, end) help the reader through each stage of the writing.

Editing

1. The author uses standard grammar throughout.
2. The author has checked spelling.
3. Punctuation aids the reader and does not have distracting errors.

Conclusion

1. The conclusion accurately sums up what the author-mathematician did.

Fig. 1 Sample mathematics writing rubric

proven in writing workshops follow. I adapted these from the work of Peter Elbow (1998) because they exemplify types of responses that teachers might typically use for longer writing and can easily be tailored to meet a variety of needs. They are deceptively simple; however, they provide valuable information to the writer and help remind participants that "responding" does not mean "correcting." For students like Laura, having a list of specific roles for listener-respondents enhances their ability to interact more efficiently and helps them get started on a positive note.

Pointing consists of naming something in the paper that was effective, interesting, creative, or particularly well done. It focuses the listener on finding an aspect of the writing that is working. Many teachers new to workshops are surprised how difficult positive response can be for students more used to negative critique. Some examples of pointing include these: "That solution addressed all the components of the task." "Your use of mathematical terminology was clear and effective." "I was surprised to find that problem could be solved using that method."

Saying back is when a respondent repeats a step or concept heard in the reading of the paper. This process is especially helpful for technically complicated problems and for getting students to see that listeners can interpret written mathematics in different ways. When responders "say back" what they heard from the writer, the writer gets an idea of whether or not the words actually conveyed the intended message or mathematical meaning. A respondent may be responsible for "saying back" the steps required in solving a problem, for example, a process that requires understanding and careful listening skills. Others may be asked to "say back" how the writer introduced or set up the problem or sum up why the conclusion was effective.

Questioning simply involves asking a question or two that the writing evoked. Some examples are "How did you conclude that the sum of the first 10 terms of the series is determined by x?" or "What was the thinking process that led to your choice of formulas?" Questions help writers focus on the effectiveness of their language choices as well as their mathematical strategies. They also help writers understand places where the writing may need clarification and further explanation.

Suggesting, a technique for more advanced students, assumes a level of proficiency or maturity in the listener. The responder suggests an idea, method, line of thinking, or approach that the writer might use to improve the work. For insecure writers, every suggestion may seem worthwhile, so use this type of response with caution.

Sharing the variety of available responses and distributing definitions can help students assume the different roles more easily.

The rubric shown in **figure 1** can easily be adapted to meet different writing needs. This one does not discuss various ability levels but reviews general categories typically a part of longer mathematical writing.

ALTERNATIVES TO READING AND RESPONDING ALOUD

There are alternatives to reading and responding aloud. Students can pass their papers to the person on their right and fill out written response sheets prepared in advance by the instructor. Feedback is given only to the paper's author (and, possibly, the instructor) so that it does not influence other respondents. As a reminder that all phases of the work are important, have less mature students attach all responses to the final draft. Also, if writers get helpful responses but do not make the effort to incorporate them in the revision stage, teachers are able to point to the need for reflection. I have students write a short reflective piece, on the back of their final drafts or in a separate paper, regarding what came easily to them, what was difficult or created problems in the writing, what they learned from respondents, and how responses effected change. Revised drafts are inevitably of better quality than first drafts, teaching students that high-stakes work is not a last-minute task. They also learn that writing about and discussing mathematics in the classroom is a valuable tool for expanding knowledge of mathematics.

SAMPLE ASSIGNMENTS

Writing can help teachers assess students' understanding of core concepts and analyze their ability to solve problems and apply knowledge. Many different types of writing can be used effectively in the mathematics classroom. The following examples provide a starting place for those wondering about what kinds of writing to try out in mathematics workshop.

1. A scientific calculator has a pi (π) key. Explain when it should be used and how it can be used effectively.
2. What is the largest area of a rectangle that has a perimeter of 48 yards? Write an explanation detailing how you determined your answer.
3. What concepts underlie the formula for the volume of a cone? For example, what physical assumptions must be made? Explain how the formula was derived and give some real-world situations for its use.
4. Define the term *congruent*. Create a list of questions that may emerge when your peers are learning about the term. Write up a dialogue between two students using your questions and providing answers to those questions. (Students could be assigned different terms.)
5. Write a paper using twenty-five advanced mathematics terms in a way that demonstrates your understanding of them. (This can also be done in a form—essay, play, poem—of your choice.)

Alternative research assignments can be useful. Most students are already familiar with essays and longer research papers, but many shorter assignments provide useful information and practice. For example, students could find a newspaper, magazine, or Internet article related to Fibonacci numbers. Have them attach the article, summarize the content, and carefully explain how the mathematical concept was integral or important. What other applications might this concept have?

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Students could write letters to mathematicians, workers in one's community, or family members, asking questions regarding mathematics concepts related to or necessary in their jobs. Students can have their peers check the letters for form and content before mailing them out.

A GOLD EXERCISE

The following scenario provides another example to demonstrate how one teacher works writing into his mathematics lessons in a variety of ways. To do so, a prompt can be used for a number of different assignments and also as a model for how mathematics teachers can create their own (Ness 2005).

Mr. Reid hands out 12 in. × 12 in. pieces of paper to each student. He asks each student to fold the paper in half repeatedly and make observations about what takes place. He then poses the following problem:

> A vault contains a large amount of gold, and you are told that you may keep as much as you can carry out, under the following conditions: On the first trip, you may only take one pound; on each successive trip, you may take out half the amount you carried out on the previous trip; you take one minute to complete each trip. Explain how much gold you can carry out and how

> long it will take to do it. Also, determine your hourly rate of earnings if you work only fifteen minutes. Use the current value of gold, which is $450 per ounce. What would be your hourly rate if you work for twenty minutes? What if you worked for an entire hour? (New York City Board of Education 1998)

All students were asked to bring their calculators to class for this particular lesson.

Students were fascinated with the observation regarding the calculation of the total amount of gold that can be carried out under the given conditions—specifically, why is it that the total amount of gold never reaches 32 ounces? The practical answer is that a little less than 2 pounds can be carried out; after about twenty-five minutes, the 2-pound figure has almost been reached, and the amounts to be carried out per trip are probably too small to measure. In fact, after just eight minutes, more than 1.99 pounds can be taken out, as can be determined by summing the terms of the sequence 1, 1/2, 1/4 , . . . , 1/128.

Many students arrived at a generalizable solution to the problem—namely, that the amount of gold in a given trip (in ounces), given the conditions mentioned above, is $16(1/2)^{(n-1)}$, where $n > 0$ and denotes any whole number of minutes or trips.

Mr. Reid motivated his students by asking them to form groups and identify real-life situations associated with this problem. He asked each group to submit a written report with drawings, photocopies, and other forms of graphics. (These reports exemplify papers that can be reviewed first in workshop.) Each group was also asked to present their project to the entire class. Mr. Reid required that his groups conduct research at the school library, science laboratory, and "civics center." The science group, for example, researched and presented a report on the half-lives of a number of elements (such as uranium and plutonium), demonstrating that the mathematical concept of the geometric series is associated with numerous phenomena related to chemistry and physics. The mathematics group made connections between the geometric series and other mathematical series—such as the harmonic series and the Fibonacci series—and the ways in which they are connected to real-world phenomena.

Can you identify five skills that were developed in the anecdote and discuss their combined importance in performing mathematical inquiry? Each of the five skills must be clearly articulated and discussed in a cohesive paragraph.

CONCLUSION

The five NCTM Process Standards are Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. These skills can be studied, reviewed, practiced, and enhanced in the well-planned forum of a mathematics writing workshop. Whether teachers implement the workshop methodology once a month, every week, or for a set number of days each term, devoting time to mathematical writing strategies is critical. Using writing in mathematical inquiry and learning discipline-related writing strategies help expand students' mathematical aptitude by demanding clarity of thinking, multiple types of expression, and control of mathematical discourse. The old rigid rules and methodological ways of teaching are substitutes for thinking (Graves 1984); the workshop format helps teachers change old stereotypes about the drudgery of mathematical writing. By necessitating social interaction and giving participants an authentic audience, writing workshops provide young mathematicians with a unique learning experience. This, in turn, fosters control of mathematical vocabulary, deeper understanding of mathematical processes, and a clear, well-organized presentation.

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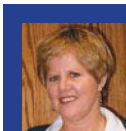

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